LRS Bianchi Type II Bulk Viscous Fluid Universe with Decaying Vacuum Energy Density Λ

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Abstract The present study deals with spatially homogeneous and locally rotationally symmetric (LRS) Bianchi type II cosmological models with bulk viscous fluid distribution of matter and decaying vacuum energy density Λ. To get the deterministic models of the universe, we assume that the expansion (θ) in the model is proportional to the shear (σ). This leads to condition $R = mS^n$, where R and S are metric potentials, m and n are constants. We have obtained two types of models of the universe for two different values of n. The vacuum energy density Λ for both models is found to be a decreasing function of time and it approaches a small positive value at late time which is supported by recent results from the observations of (SN Ia). Some physical and geometric behaviour of these models are also discussed.

1 Introduction

The present day universe is satisfactorily described by homogeneous and isotropic models given by the FRW space-time. The universe in a smaller scale is neither homogeneous nor isotropic nor do we expect the Universe in its early stages to have these properties. Recently, experimental studies of isotropy of the Cosmic Microwave Background Radiation (CMBR) and speculation about the amount of helium formed at early stages and other effects also have stimulated much theoretical interest in anisotropic cosmological models. Although the standard cosmological models have a very good agreement with present day universe but do not give a clear description of the early universe. A physically realistic description is best given by inhomogeneous models. The spatially homogeneous and anisotropic Bianchi type models present a "middle way" between FRW models and inhomogeneous

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and anisotropic universes and hence play an important role in modern cosmology. Homogeneous and anisotropic cosmological models have been widely studied in the framework of General Relativity in the search of a realistic picture of the universe in its early stages.

Bianchi type-II space-time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of universe. Asseo and Sol [1] emphasized the importance of Bianchi type-II universe. A spatially homogeneous Bianchi model necessarily has a three-dimensional group, which acts simply transitively on space-like three-dimensional orbits. Here we confine ourselves to a locally rotationally symmetric (LRS) model of Bianchi type-II. This model is characterized by three metric functions $R_1(t)$, $R_2(t)$ and $R_3(t)$ such that $R_1 = R_2 \neq R_3$. The metric functions are functions of time only. (For non-LRS Bianchi metrics we have $R_1 \neq R_2 \neq R_3$). For LRS Bianchi type-II metric, Einstein's field equations reduce, in the case of perfect/bulk viscous fluid distribution of matter, to three nonlinear differential equations. Bali et al. [2] have investigated an LRS Bianchi type II space-time filled with string dust fluid and the shear (σ) is proportional to the expansion (θ). Several authors [3–13] have studied LRS Bianchi type II cosmological models in various contexts.

In general relativity, the cosmological constant Λ may be regarded as a measure of the energy density of the vacuum, and can in principle lead the avoidance of the big-bang singularity which is characteristic of other Friedmann-Robertson-Walker (FRW) models. However, the rather simplistic properties of the vacuum that follow from the usual form of Einstein's equations can be made more realistic if that theory is extended, which in general leads to a variable Λ. There are significant observational evidence for the detection of Einstein's cosmological constant Λ or a component of material content of the universe that varies slowly with time and space to act like Λ. Some of the recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmological constant by Ratra and Peebles [13], Sahni and Starobinsky [14] point out that in the absence of any interaction with matter or radiation, the cosmological constant remains a "constant". However, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. This entails that energy has to be conserved by a decrease in the energy density of the vacuum component followed by a corresponding increase in the energy density of matter or radiation (see also Alam et al. [15, 16], Carroll, Press and Turner [17], Peebles [18], Padmanabhan [19, 20]). There is a plethora of astrophysical evidence today, from supernovae measurements (Perlmutter et al. [21–23], Riess et al. [24–27], Garnavich et al. [28, 29], Schmidt et al. [30], Blakeslee et al. [31], Astier et al. [32]), the spectrum of fluctuations in the Cosmic Microwave Background (CMB) [33], baryon oscillations [34] and other astrophysical data, indicating that the expansion of the universe is currently accelerating. The energy budget of the universe seems to be dominated at the present epoch by a mysterious dark energy component, but the precise nature of this energy is still unknown. Many theoretical models provide possible explanations for the dark energy, ranging from a cosmological term [35, 36] to super-horizon perturbations [37, 38] and time-varying quintessence scenarios [39]. These recent observations strongly favour a significant and a positive value of Λ with magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. In Ref. [26], Riess et al. have recently presented an analysis of 156 SNe including a few at $z > 1.3$ from the Hubble Space Telescope (HST) "GOOD ACS" Treasury survey. They conclude to the evidence for present acceleration $q_0 < 0$ ($q_0 \approx -0.7$). Observations (Knop et al. [40]; Riess et al., [26]) of Type Ia Supernovae (SNe) allow us to probe the expansion history of the universe leading to the conclusion that the expansion of the universe is accelerating.

Most studies in cosmology involve a perfect fluid. Large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we

should analyze dissipative effects in cosmology. Further, there are several processes which are expected to give rise to viscous effect. These are the decoupling of neutrinos during the radiation era and the recombination era [41], decay of massive super string modes into mass-less modes [42], gravitational string production [43, 44] and particle creation effect in grand unification era [45]. It is known that the introduction of bulk viscosity can avoid the big bang singularity. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [46] for a review on cosmological models with bulk viscosity). A uniform cosmological model filled with fluid which possesses pressure and second (bulk) viscosity was developed by Murphy [47]. The solutions that he found exhibit an interesting feature that the big bang type singularity appears in the infinite past. Bali and Pradhan [48] have investigated Bianchi type III string cosmological models with time dependent bulk viscosity. Bali and Singh [49] have studied Bianchi type V bulk viscous fluid string dust cosmological model in general relativity. Bali [50] also obtained magneto-viscous fluid cosmological model of plane symmetry in general relativity. Pradhan et al. [51–57] and Singh et. al. [58] have studied bulk viscous cosmological models with varying Λ-term in different space-times.

Recently, Bali and Banerjee [59] have obtained LRS Bianchi type II cosmological models for perfect fluid distribution in general relativity. Motivated by these investigations, we have revisited the above solutions [55] and generalized these solutions for better results. Here we wish to approach this subject from a different prospective, and present a new class of exact solutions wherein cosmological term Λ can change in a manner that is in agreement with observations. In this paper, we have investigated LRS Bianchi type-II cosmological models with varying cosmological constant in presence of bulk viscous fluid in an expanding universe. The plan of the paper is as follows. The metric and the field equations are presented in Sect. 2. In Sect. 3, we deal with the solutions of the field equations in general form. In Sect. 4, we have derived first model by considering $n = \frac{1}{2}$. We have also discussed the physical and geometric features of this model. In Sect. 5, we have derives second model by considering $n = \frac{3}{2}$ and also discussed the properties of this model. In Sect. 6, we have given the concluding remarks.

2 The Metric and Field Equations

We consider LRS Bianchi type metric in the form

$$ds^2 = \eta_{ab}\theta^a\theta^b, \tag{1}$$

where

$$\theta^1 = Rdx, \qquad \theta^2 = S(dy - xdz), \qquad \theta^3 = Rdz, \qquad \theta^4 = dt. \tag{2}$$

Thus, the metric (1) leads to

$$ds^2 = -dt^2 + R^2dx^2 + S^2(dy - xdz)^2 + R^2dz^2, \tag{3}$$

where R and S are functions of cosmic time t only.

The spatial volume of this model is given by

$$V^3 = R^2S. \tag{4}$$

Here, we also define $V = (R^2 S)^{\frac{1}{3}}$ as the average scale factor so that Hubble's parameter is defined by

$$H = \frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}\right), \tag{5}$$

where an overdot denotes differentiation with respect to the cosmic time t.

The energy momentum tensor for bulk viscous fluid distribution is taken as

$$T_i^j = (\rho + \bar{p})v_i v^j + \bar{p} g_i^j, \tag{6}$$

where

$$\bar{p} = p - \xi v_{;i}^i. \tag{7}$$

Here ρ, p, $\bar{p}$ and ξ are energy density, isotropic pressure, effective pressure and bulk viscous coefficient respectively and $v^i = \frac{dx^i}{ds}$ is the four-velocity satisfying the condition

$$g_{ij}v^i v^j = -1. \tag{8}$$

We assume that coordinates to be comoving so that $v^1 = 0 = v^2 = v^3$, $v^4 = 1$.

The Einstein's field equations (in geometrized units for which $c = 1$, $G = 1$)

$$R_i^j - \frac{1}{2}R g_i^j + \Lambda g_i^j = -8\pi T_i^j, \tag{9}$$

for the line-element (1) lead to

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3}{4}\frac{S^2}{R^4} + \Lambda = -8\pi \bar{p}, \tag{10}$$

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} + \Lambda = -8\pi \bar{p}, \tag{11}$$

$$\frac{2\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2}{R^2} - \frac{1}{4}\frac{S^2}{R^4} - \Lambda = 8\pi\rho. \tag{12}$$

3 Solutions of the Field Equations

The field equations (10)–(12) are a system of three equations with five unknown parameters R, S, p, ρ and Λ. Two additional constraints relating these parameters are required to obtain explicit solutions of the system. We assume that the expansion (θ) in the model is proportional to the shear (σ). This condition leads to

$$R = m S^n, \tag{13}$$

where m and n are constants and

$$\theta = \frac{2\dot{R}}{R} + \frac{\dot{S}}{S}, \tag{14}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{R}}{R} - \frac{\dot{S}}{S}\right). \tag{15}$$

The motive behind assuming this condition is explained with reference to Thorne [60], the observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 per cent [61, 62]. To put more precisely, red-shift studies place the limit

$$\frac{\sigma}{H} \leq 0.3$$

on the ratio of shear σ to Hubble constant H in the neighbourhood of our Galaxy today. Collins et al. [63] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

To solve the field equations, we follow the technique of Bali and Banerjee [59]. From (10) and (11), we obtain

$$\frac{\dot{R}\dot{S}}{RS} - \frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{S^2}{R^4} - \frac{\dot{R}^2}{R^2} = 0. \tag{16}$$

Equations (13) and (16) lead to

$$\frac{\ddot{S}}{S} + 2n\left(\frac{\dot{S}}{S}\right)^2 = \frac{1}{m(n-1)}S^{2(1-2n)}, \tag{17}$$

which reduces to

$$2\ddot{S} + 4n\frac{\dot{S}^2}{S} = \frac{2}{m(n-1)}S^{(3-4n)}. \tag{18}$$

Equation (18) takes the form as

$$\frac{d}{dS}(f^2) + \frac{4n}{S}f^2 = \frac{2}{m(n-1)}S^{(3-4n)}, \tag{19}$$

where $\dot{S} = f(S)$, $\ddot{S} = ff'$, $f' = \frac{df}{dS}$.

From (19), we obtain

$$f^2 = \dot{S}^2 = \frac{1}{2m(n-1)}S^{4(1-n)} + kS^{-4n}, \tag{20}$$

k being an integrating constant. Equation (20) leads to

$$\frac{S^{2n}dS}{\sqrt{\frac{1}{[2m(n-1)S^4+k]}}} = dt. \tag{21}$$

The solution of (21) is not tenable for $n = 1$. One can choose the value of n such that above relation be integrable. We consider the following two cases.

4 First Model

When $n = \frac{1}{2}$, (23) reduces to

$$\frac{\sqrt{m}SdS}{\sqrt{mk - S^4}} = dt, \tag{22}$$

which after integration leads

$$S^2 = \sqrt{mk} \sin\left\{ \frac{2}{\sqrt{m}}(t+\alpha) \right\}, \tag{23}$$

where α is a constant of integration. Equation (13) for $n = \frac{1}{2}$, leads to

$$R^2 = m^{\frac{5}{4}} k^{\frac{1}{4}} \sqrt{\sin\left\{ \frac{2}{\sqrt{m}}(t+\alpha) \right\}}. \tag{24}$$

Therefore, the metric (3) reduced to

$$ds^2 = -dT^2 + m^{\frac{5}{4}} k^{\frac{1}{4}} \sqrt{\sin\left\{ \frac{2T}{\sqrt{m}} \right\}}(dx^2 + dz^2) + \sqrt{mk} \sin\left\{ \frac{2T}{\sqrt{m}} \right\}(dy - xdz)^2, \tag{25}$$

where $t + \alpha = T$.

The expressions for effective pressure ($\bar{p}$) and density (ρ) for the model (25) are given by

$$8\pi\bar{p} = \frac{5}{4m} \cot^2\left(\frac{2T}{\sqrt{m}} \right) + \frac{3}{m} - \frac{1}{4m^2} - \Lambda, \tag{26}$$

$$8\pi\rho = \frac{1}{m} \cot^2\left(\frac{2T}{\sqrt{m}} \right) - \frac{1}{4m^2} + \Lambda. \tag{27}$$

For the specification of ξ, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{28}$$

where γ $(0 \leq \gamma \leq 1)$ is a constant. Thus, given $\xi(t)$ we can solve for the cosmological parameters. In most of the investigation involving bulk viscosity it is assumed to be a simple power function of the energy density [64–69]

$$\xi(t) = \xi_0 \rho^\beta, \tag{29}$$

where ξ_0 and β are constants. For small density, β may even be equal to unity as used in Murphy's work [47] for simplicity. If $\beta = 1$, (29) may correspond to a radiative fluid [69]. Near a big bang, $0 \leq \beta \leq \frac{1}{2}$ is a more appropriate assumption [70] to obtain realistic models.

For simplicity and realistic models of physical importance, we consider the following two cases ($\beta = 0, 1$).

On using (29) and (7) in (26), we obtain

$$8\pi(p - \xi_0\rho^\beta\theta) = \frac{5}{4m} \cot^2\left(\frac{2T}{\sqrt{m}} \right) + \frac{3}{m} - \frac{1}{4m^2} - \Lambda, \tag{30}$$

where θ is the scalar of expansion calculated for the flow vector v^i and is given by

$$\theta = \frac{2}{\sqrt{m}} \cot\left(\frac{2T}{\sqrt{m}} \right). \tag{31}$$

4.1 Model I: Solution when $\beta = 0$

When $\beta = 0$, (29) reduces to $\xi = \xi_0$. With the use of (27), (28) and (31), (30) reduces to

$$8\pi(1 + \gamma)\rho = \frac{9}{4m}\cot^2\left(\frac{2T}{\sqrt{m}}\right) + \frac{16\pi\xi_0}{\sqrt{m}}\cot^2\left(\frac{2T}{\sqrt{m}}\right) - \frac{1}{2m^2} + \frac{3}{m}. \qquad (32)$$

Eliminating $\rho(t)$ between (27) and (32), we obtain

$$(1 + \gamma)\Lambda = \frac{(5 - 4\gamma)}{4m}\cot^2\left(\frac{2T}{\sqrt{m}}\right) + \frac{16\pi\xi_0}{\sqrt{m}}\cot\left(\frac{2T}{\sqrt{m}}\right) - \frac{1}{4m^2} + \frac{3}{m}. \qquad (33)$$

4.2 Model II: Solution when $\beta = 1$

When $\beta = 1$, (29) reduces to $\xi = \xi_0\rho$. With the use of (27), (28) and (31), (30) reduces to

$$8\pi\left[\gamma - \frac{2\xi_0}{\sqrt{m}}\cot\left(\frac{2T}{\sqrt{m}}\right)\right]\rho = \frac{9}{4m}\cot^2\left(\frac{2T}{\sqrt{m}}\right) - \frac{1}{2m^2} + \frac{3}{m}. \qquad (34)$$

Eliminating $\rho(t)$ between (27) and (34), we obtain

$$\left[\gamma - \frac{2\xi_0}{\sqrt{m}}\cot\left(\frac{2T}{\sqrt{m}}\right)\right]\Lambda$$

$$= \frac{9}{4m}\cot^2\left(\frac{2T}{\sqrt{m}}\right) - \frac{1}{2m^2} + \frac{3}{m}$$

$$- \left\{\frac{1}{m}\cot^2\left(\frac{2T}{\sqrt{m}}\right) - \frac{1}{4m^2}\right\}\left[\gamma - \frac{2\xi_0}{\sqrt{m}}\cot\left(\frac{2T}{\sqrt{m}}\right)\right] \qquad (35)$$

4.3 Physical and Geometric Features

From (32) and (34), we note that $\rho(t)$ is a decreasing function of time and $\rho > 0$ for $m \geq \frac{1}{4}$.

Now a days, the cosmological Λ-term has attracted theoreticians and observers for scientific elegance. The nontrivial role of the vacuum in the early universe generated a Λ-term that has led to inflationary phase. Observationally, this term provides an additional parameter to accommodate conflicting data on the values of the Hubble constant, the deceleration parameter, the density parameter and the age of the universe (for example, see the references [71, 72]). The behaviour of the universe in this model will be determined by the cosmological term Λ; this term has the same effect as a uniform mass density $\rho_{eff} = -\Lambda/4\pi G$, which is constant in space. A positive value of Λ corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe with a positive value of Λ, the expansion will tend to accelerate; whereas in the universe with negative value of Λ, the expansion will slow down, stop and reverse. From (32), we see that for $m \geq \frac{1}{4}$ the cosmological term Λ is a decreasing function of time and it approaches a small positive value at late time. Recent cosmological observations (Garnavich et al. [28, 29], Perlmutter et al. [21–23], Riess et al. [24–27], Schmidt et al. [30]) suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus, our model is consistent with the results of recent observations.

With regard to the kinematic properties of the velocity vector v^i for the model (25), a straight forward calculation leads to the expressions for the Hubble's parameter, shear σ, deceleration parameter q and proper volume V^3 of the fluid:

$$H = \frac{2}{3\sqrt{m}} \cot\left(\frac{2T}{\sqrt{m}}\right), \tag{36}$$

$$\sigma = \frac{1}{2m\sqrt{3}} \left|\cot\left(\frac{2T}{\sqrt{m}}\right)\right|, \tag{37}$$

$$q = -\frac{V\ddot{V}}{\dot{V}^2} = -1 + 3\sqrt{m}\sec^2\left(\frac{2T}{\sqrt{m}}\right), \tag{38}$$

$$V^3 = \sqrt{-g} = m\sqrt{mk}\sin\left(\frac{2T}{\sqrt{m}}\right). \tag{39}$$

From (38), we observe that

$$q < 0 \quad \text{if } \sec^2\left(\frac{2T}{\sqrt{m}}\right) < \frac{1}{3\sqrt{m}},$$

and

$$q > 0 \quad \text{if } \sec^2\left(\frac{2T}{\sqrt{m}}\right) > \frac{1}{3\sqrt{m}}.$$

The model (25) starts with a big bang at $T = 0$ and the expansion in the model decreases as time increases. However, the expansion in the model stops when $T = \frac{\pi\sqrt{m}}{4}$. The model, in general, represents an expanding, shearing and non-rotating universe. The spatial volume increases as time increases. There is a Point Type singularity in the model at $T = 0$ (MacCallum [73]). For the condition $\sec^2\left(\frac{2T}{\sqrt{m}}\right) < \frac{1}{3\sqrt{m}}$, the solution gives accelerating model of the universe. It can be easily seen that when $\sec^2\left(\frac{2T}{\sqrt{m}}\right) > \frac{1}{3\sqrt{m}}$, our solution represents decelerating model of the universe. It is remarkable to mention here that the model (25) involves periodic functions and gives rise to cyclic mode of expansion.

5 Second Model

When $n = \frac{3}{2}$, (21) reduces to

$$\frac{\sqrt{m}\,S^3\,dS}{\sqrt{S^4 + mk}} = dt, \tag{40}$$

which, after integration, leads to

$$S^4 = \frac{4}{m}(t + K)^2 - mk, \tag{41}$$

where K is the constant of integration. Equation (13) for $n = \frac{3}{2}$ leads to

$$R^2 = m^2 S^3 = m^2\left[\frac{4(t + K)^2}{m} - mk\right]^{\frac{3}{4}}. \tag{42}$$

Therefore, the metric (3) reduces to the form

$$ds^2 = -dT^2 + m^{\frac{5}{4}}(4T^2 - m^2k)^{\frac{3}{4}}(dx^2 + dz^2) + \frac{1}{\sqrt{m}}(4T^2 - m^2k)^{\frac{1}{2}}(dy - xdz)^2, \quad (43)$$

where $t + K = T$.

The expressions for effective pressure ($\bar{p}$) and density (ρ) for the model (43) are given by

$$8\pi\bar{p} = \frac{T^2 + 5m^2k}{(4T^2 - m^2k)^2} - \frac{1}{4m^3(4T^2 - m^2k)} - \Lambda, \quad (44)$$

$$8\pi\rho = \frac{21T^2}{(4T^2 - m^2k)^2} - \frac{1}{4m^3(4T^2 - m^2k)} + \Lambda. \quad (45)$$

On using (29) and (7) in (44)

$$8\pi(p - \xi_0\rho^\beta\theta) = \frac{21T^2}{(4T^2 - m^2k)^2} - \frac{1}{4m^3(4T^2 - m^2k)} + \Lambda, \quad (46)$$

where θ is the scalar of expansion calculated for the flow vector v^i and is given by

$$\theta = \frac{8T}{(4T^2 - m^2k)}. \quad (47)$$

5.1 Model I: Solution when $\beta = 0$

When $\beta = 0$, (29) reduces to $\xi = \xi_0$. With the use of (45), (28) and (47), (46) leads to

$$8\pi(1 + \gamma)\rho = \frac{22T^2 + 5m^2k}{(4T^2 - m^2k)^2} + \frac{64\pi\xi_0 T}{(4T^2 - m^2k)} - \frac{1}{2m^3(4T^2 - m^2k)}. \quad (48)$$

Eliminating $\rho(t)$ between (45) and (48), we obtain

$$(1 + \gamma)\Lambda = \frac{(1 - 21\gamma)T^2 + 5m^2k}{(4T^2 - m^2k)^2} + \frac{64\pi\xi_0 T}{(4T^2 - m^2k)} - \frac{(1 - \gamma)}{4m^3(4T^2 - m^2k)}. \quad (49)$$

5.2 Model II: Solution when $\beta = 1$

When $\beta = 1$, (29) reduces to $\xi = \xi_0\rho$. With the use of (45), (28) and (47), (46) leads to

$$8\pi\left[1 + \gamma - \frac{8\xi_0 T}{(4T^2 - m^2k)}\right]\rho = \frac{22T^2 + 5m^2k}{(4T^2 - m^2k)^2} - \frac{1}{2m^3(4T^2 - m^2k)}. \quad (50)$$

Eliminating $\rho(t)$ between (45) and (50), we obtain

$$\begin{aligned}
\left[1 + \gamma - \frac{8\xi_0 T}{(4T^2 - m^2k)}\right]&\Lambda \\
&= \frac{22T^2 + 5m^2k}{(4T^2 - m^2k)^2} - \frac{1}{2m^3(4T^2 - m^2k)} \\
&\quad - \left[1 + \gamma - \frac{8\xi_0 T}{(4T^2 - m^2k)}\right]\left\{\frac{21T^2}{(4T^2 - m^2k)^2} - \frac{1}{4m^3(4T^2 - m^2k)}\right\}. \quad (51)
\end{aligned}$$

5.3 Physical and Geometric Features

The reality of energy density depends on the values of constants m and k. From (48) and (50), we observe that the energy density is a decreasing functions of time and it is always positive. From (49) and (51), we observe that under suitable consideration of constants k and m, the cosmological term Λ decreases as time increases and it approaches to small positive value at late time. This is a good agreement with the recent observations already discussed in Sect. 4.3.

With regard to the kinematic properties of the velocity vector v^i for the model (43), we obtain the expressions for the Hubble's parameter, shear σ, deceleration parameter q and proper volume V^3 of the fluid as

$$H = \frac{8T}{3(4T^2 - m^2k)}, \tag{52}$$

$$\sigma = \frac{T}{\sqrt{3}(4T^2 - m^2k)}, \tag{53}$$

$$q = \frac{1}{2} + \frac{3m^2k}{8T^2}, \tag{54}$$

$$V^3 = m(4T^2 - m^2k). \tag{55}$$

The model (43) starts with a big bang at $T = \frac{m\sqrt{k}}{2}$ and the expansion in the model decreases as time increases. In general, the model represents an expanding, shearing and non-rotating universe. The spatial volume increases as time increases. There is a Point Type singularity in the model at $T = \frac{m\sqrt{k}}{2}$ (MacCallum [73]).

6 Concluding Remarks

We have obtained a new class of LRS Bianchi type II cosmological models in presence of bulk viscous fluid distribution of matter with decaying vacuum energy. We have presented an alternative and straightforward approach to solve the Einstein's typical, non-linear field equations by considering the expansion in the model is proportional to the shear as Collins et al. [63] have showed that the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta} = $ constant. Both models (25) and (43) start with a big bang singularity. We have obtained a Point Type singularity in both models. In general, the models represent an expanding, shearing and non-rotating universe.

In both models discussed in Sects. 4 and 5, the cosmological term Λ is found to be a decreasing function of time and it approaches a small positive value at late time which is supported by recent results from the observations of Type Ia supernova explosion (SN Ia). Naturally a cosmological model is required to explain acceleration in the present universe. Thus, our theoretical models are consistent with the results of recent observations.

The effect of bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. The effect is clearly visible on the p effective (see details in previous sections). We have shown regular well behaviour of energy density, cosmological term (Λ) and the expansion of the universe with parameter T. We also observe that Murphy's conclusions [47] about the absence of a big bang type singularity in the infinite past in models with bulk viscous fluid, in general, is not true. The results obtained by Myung and Cho [42] also showed that, it is, in general, not valid, since for some cases

big bang singularity occurs in finite past. Our solutions generalize the solutions recently obtained by Bali and Banerjee [59].

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